Designing an intuitive interface to enhance trigonometry learning

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doi: 10.33114/adim.2019.03.216

In the last three decades, the application of TUIs (tangible user interfaces) in education has demonstrated its positive influence on performance and learning of students. At Universidad del Desarrollo in Chile, monitoring of diagnostic tests over time evidences difficulties and challenges in the teaching-learning of trigonometry in first-year Engineering education. This study consisted in designing and validating a tangible interface to learn trigonometry in the classroom setting. The methodology used was a quasi-experiment with first-year students from the Schools of Design and Engineering at Universidad del Desarrollo in Chile. Principles of the theory of Embodied Cognition and Blended Interaction were applied to model an intuitive, collaborative and meaningful learning experience. During the design process, three Intermediate Models were tested with several types of users, and two Prototypes were tested with an experimental group. User-testing highly contributed to the design of the interaction experience and the interface, progressively defining the functional and pedagogical aspects. Comparative analysis of Pre and Post-Test results, demonstrate that students’ performance increased by 37.1% after two sessions using the interface.

Keywords: Design, learning, tangible user Interfaces, interaction, trigonometry

Introduction

Traditional education systems have rapidly changed in the last twenty years, incorporating not only more collaborative and inclusive methodologies but also new tools and technologies that encourage participation and interaction. This evolution has gone from the development of software applications, websites, etc. (Camilleri & Camilleri, 2017; Dooley, Ellison, Welch, Allen, & Bauer, 2016; van Loon, Ros, & Martens, 2012), to learning interfaces that provide direct and "tactile" interaction between the user and the interface. Numerous educational initiatives have relied on the benefits of tangible user interfaces (TUIs), especially in primary education, demonstrating the variety of its possible uses. Nevertheless, in mathematics education, and more specifically in the teaching-learning of trigonometry, there is an unexplored potential or gap in the use of tangibility to understand and relate abstract concepts (Marshall, 2007). For example Geogebra, a software to learn mathematics (https://www.geogebra.org) enables simultaneous visual, graphic and numerical representations of mathematical objects that are explored interactively, supporting an intuitive “learning by doing” approach (Kepeçoğlu & Yavuz, 2016). However, it is mainly supported only by vision and the corporal interaction is reduced to the individual use of the mouse. The interface designed in this study used the benefits of TUIs in combination with the intuitive and visual learning aspects of tools such as Geogebra. This combination enables to have low entry limits, use multiple senses, approach intuitively, and encourage collaborative learning through social interaction.
Purpose of the research

The main objective of this study was to investigate the formal, perceptual and technological variables to be considered in the design of a tangible educational interface to learn basic concepts of trigonometry.

Four specific objectives were defined:

13. Determine the content and progression of the didactic experience to facilitate learning through an interactive interface.
14. Iteratively design a multisensory interface to facilitate the understanding of concepts of trigonometry in higher education students.
15. Investigate the optimal interactions needed to operate the interface correctly and understand the subject properly.
16. Measure the impact of the experience on student learning.

Theoretical framework

Tangible User Interfaces

Tangible User interfaces (TUIs) are human-computer interfaces that give physical form to digital elements, enabling the user to directly manipulate digital information with their hands (Ishii & Ullmer, 1997). The first approaches to TUIs appeared at the end of the nineties, attributable to the seminal work of Irôshi Ishii in the Tangible Media Group of MIT. Today, TUIs are increasingly accessible (due to the democratization of technologies) and its fields of application have diversified into the public and private sphere: kiosks in science museums, pieces of interactive art, collaborative tables in offices, and more recently in wearable interfaces. In the field of education, several examples can be found in material designed for young children, such as Cubetto (https://www.cubetto.com), assemblable blocks to learn the basics of programming.

The use of TUIs positively influences the cognitive process of the user, enabling agile opportunities of individual and collaborative discovery, supporting the formation of hypotheses, improving understanding through the combination of different senses and supporting the rapid evaluation of diverse alternatives (Jetter, Reiterer & Geyer, 2014). TUIs influence the thought processes supported by experimentation (Resnick et al., 2005) and enable a better use of haptic resources that we develop when manipulating everyday objects that surround us (Ishii, 2007). All of the features described above make TUIs suitable to enable natural and multisensory learning experiences.

This study was centered in the design of a TUI capable of hosting collaboration among multiple participants, who build their interaction experience using and enhancing their pre-existing skills (motor, spatial, social and cognitive). By escaping from the traditional interaction paradigms of the so-called WIMP interfaces -Windows, Icons, Menu, Pointer- (Gentner & Nielson, 1996), the interface fosters embodied, collaborative interactions and activates cognitive processes stimulated and facilitated by design.

Embodied Cognition, Conceptual Metaphors and Blended Interaction

The designed interface focused on concretizing abstract mathematical concepts through the manipulation of tangible components, appealing to the transfer of conceptual metaphors. Mathematical conceptual knowledge is largely metaphorical, making frequent use of conceptual transfer, as when we conceptualize numbers as points in a line. Since many of our concepts are metaphorical, our conceptual understanding is crucially dependent on the nature of our bodies and the physical environment in which they function (Lakoff, 2009).

The Theory of Embodied Cognition states that the way in which we learn new concepts is based on the previous body of knowledge that we store since childhood. Lakoff and Núñez (2000), explain that human beings internalize abstract concepts in concrete terms using ideas and modes of reasoning based on the sensorimotor system. The mechanism by which an abstract concept is understood in concrete terms is called Conceptual Metaphor, which consists of a transfer or “mapping” of a “source concept” to another “object concept” (target). The source concepts are often concrete and have some kind of “body base” (Johnson, 2013; Dodge & Lakoff, 2005; Feldman, 2008; Pecher, Boot, & Van Dantzig, 2011), while the object concepts are often abstract and cannot be directly experienced or perceived. In the pedagogical context of this project, metaphors can serve to convey a basic understanding of abstract notions of mathematics and to encapsulate
them in a vivid and informative "image", thus facilitating the internalization of abstract concepts (Font, Bolite, & Acevedo, 2010).

The present study took the approach proposed by Jetter, Reiterer and Geyer (2014), which links Embodied Cognition with the HCI field, and names it “Blended Interaction”. The cited authors introduce the term Conceptual Blends to explain how users of an interface rely on familiar concepts and the real world when they are exposed to new experiences of digital technology. The mixtures create a new concept from two inputs (cognitive and physical), in this way, the output results in a new concept with a new emerging structure that was previously not available in the inputs. Several authors have postulated that the appropriate combination of previous experiences of the physical and social world, in combination with those already familiar in the digital domain, enable an interface to be understood and operated more naturally (Hutchins, Hollan & Norman, 1985; Jetter et al., 2014). This enables compromising few cognitive resources to connect evaluation and execution when facing a new system. To achieve this objective, four dimensions defined by the authors Jetter et al. (2014) were considered as fundamental variables to design the interface:

- Individual Interaction: focused on individual capabilities supported by multimodal elements.
- Social interaction: group activities subject to pre-established conventions that come from daily life interactions.
- User Journey or Workflow: definition of a journey that is temporarily ordered constituting the experience.
- Physical Space: the form-function of the interface components and the layout of the physical space to define the nature of the experience.

**Teaching trigonometry**

In traditional teaching, students are subjected to repetitive procedures (Skinner, 1976), and learning is assessed through cyclical evaluations. The research team believes that this type of learning does not usually stimulate creativity or incorporate the premise that human beings add meanings to the thought process (Johnson, 2013). Thus, in this study, the design of the experience is sustained in a constructivist vision: students learn by constructing something with their prior knowledge (Ambrose, Bridges, Dipietro, Lovett, & Norman, 2010) in combination with new input information to produce significant learning (Vygotsky, 1980). In this approach, students are challenged with a problem and encouraged to work on their own to solve it, promoting higher levels of thinking.

Trigonometry is a fundamental requirement for the study of both advanced mathematics and science. In the United States, the National Council of Mathematics Teachers highlights the importance of trigonometry in the study of periodic functions and emphasizes the usefulness of trigonometry in the investigation of real-world phenomena (Curri, 2012). Moreover, trigonometry is fertile in connecting algebraic, geometric and graphic reasoning (Weber, 2005). Nevertheless, despite its importance in both high school and university, research shows that trigonometry remains a difficult subject for both students and teachers (Brown, 2006; Thompson, Carlson & Silverman, 2007; Weber, 2005).

At Universidad del Desarrollo, Santiago, Chile, high-school level difficulties in trigonometry are evidenced by the results of the diagnostic test applied to first-year Engineering students at the beginning of their first semester. Results have shown systematically for several years that trigonometry is a subject that presents comparative deficiencies. As seen on Table 1, in 2017, trigonometry presented the lowest performance (14%) within all the subjects addressed in the diagnostic test.

<table>
<thead>
<tr>
<th>Thematic Area</th>
<th>Total number of answers</th>
<th>Total correct answers</th>
<th>Global performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter areas of simple figures</td>
<td>916</td>
<td>300</td>
<td>33%</td>
</tr>
<tr>
<td>Layout and orientation of straight lines in the plane</td>
<td>687</td>
<td>279</td>
<td>41%</td>
</tr>
<tr>
<td>Equations and quadratic functions</td>
<td>229</td>
<td>108</td>
<td>47%</td>
</tr>
</tbody>
</table>

Table 1: 2017 Diagnostic evaluation results of the career of Engineering at Universidad del Desarrollo, Santiago, Chile
Methodology

The methodology of this study is quasi-experimental and mixed, incorporating qualitative and quantitative data collection to inform the development of the design from the perspective of usability and pedagogical effectiveness (Gilbert & Driscoll, 2002; Jetter et al., 2014). The design process was organized using a modified version of the Double Diamond Model created by the Design Council (2014). It considers an iterative process of four convergent and divergent thought cycles (discover, define, develop, deliver). The divergent cycles consisted of literature review, characterization of intermediate models and prototypes, and exploration of design options for the interface. In the convergent phases Intermediate Models and Prototypes were tested balancing user feedback with observation of participatory processes (Resnick et al., 2005), based on two co-dependent factors: the content to be transmitted and the integration of visual, auditory, spatial and interactive components.

The first set of prototypes (“Intermediate Models”) consisted in a sequence of three low-fidelity models that were tested to explore particular aspects of interaction. In the following cycles, two higher-fidelity prototypes (“Prototype 1 and 2”) were tested to validate the design decisions taken in the preceding cycle. The two user-testing phases with Prototypes 1 and 2 involved thirty minute-long interactions, guided by a Facilitator who used a pre-established script. Each session was documented in video, photography and an observation spreadsheet developed by the researchers. Additionally, in each session a usability and interface validation questionnaire was applied using a Likert scale from 1 to 9 with criteria taken from the VII version of the Questionnaire for User Interaction Satisfaction (Chin, Diehl, & Norman, 1988).

Results of a Pre and Post Test were compared to measure the impact of the interface on students’ learning. Two groups were established consisting of 119 first-year students from the Design and Engineering careers who carried out the Pre-Test (n=65 in Design and n=54 in Engineering). Within these groups, 31 students (n=13 Design and n=18 Engineering) tested the first prototype, and from that group, 24 subjects tested the second prototype (n=10 Design and n=14 Engineering) and constitute the Experimental Group. The rest of the students constituted the Control Group and carried out the Post-Test without interacting with the interface. The study also considers a third group called "Model testers", students and teachers from both careers who qualitatively tested the Intermediate Models. Figure 2 shows the activities for each group in time.

Figure 1: Adaptation of the Double Diamond model Design Council (2014) that illustrates the interface design process.
Designing the Experience and Interface

To some extent, the flaws in trigonometry learning are due to the fact that students resort to memorization when facing a trigonometric problem (Méndez & Leal, 2018). Studies reveal that learning through the memorization of both concepts and procedures does not lead to a real understanding of the subject and generally only point to the quick solution of a mathematical problem, leaving aside the conceptual understanding and the ability to understand the same concept represented in different ways (Weber, 2005). In contrast, the designed interface considers a multimodal approach (vision, hearing and touch in tandem), thus providing different ways to access the information. The pedagogical experience designed in this study aimed to promote higher levels of thought (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956; Driscoll, 2000) to activate previous knowledge and/or correct misconceptions and integrate them to the new concepts (Barry, Kanematsu, Kobayashi, & Shimofuruya, 2003).

Pedagogical dimension

At early stages of the design process, five thematic modules were defined to determine the scope of content to be addressed. Having a content framework provided a roadmap for design development and enabled to conceive a coherent pedagogical experience supported by the interface. The contents were defined based on the Pre-Tests, literature review and the previous experience of Engineering professors. The five modules are:

- Module 1: Cartesian plane and polar coordinates
- Module 2: Sine values based on an angle
- Module 3: Cosine values based on an angle
- Module 4: Periodic functions and sine waves
- Module 5: Special angles and trigonometric identities

The modules are not necessarily presented in sequence during the experience. To concretize the order of presentation of concepts, a script organized in twelve sequential “scenes” was created to provide a structure for the experience. The script progresses constructively, from the most simple to the most complex. For instance, the first scene is restricted to only understand how the Y axis of the cartesian plane works, and the final scene is about exploring periodic functions and understanding their applications in the physics world. The leading principle that drives the experience is that students are challenged with an open question and encouraged to work collaboratively on their own to find the answer, abilitating students to have clear spaces to err, experiment, explore and discover. It also provides a general guidance for the facilitator that hosts the sessions in respect to the moment in which to ask the participants for specific interactions, what questions to ask, and determining the milestones that require an assessment of the understanding level.

Multisensory dimension

Following one of the principles to create effective interfaces mentioned by Resnick et al. (2005), the interface was designed to provide low limits, high ceilings and wide walls: An effective interface enables novices to participate (low limits), offers experienced higher level activities (high ceiling) and offers a wide range of exploration possibilities (wide walls). In addition, the interface design was built upon the benefits of integrating multiple peripheral senses (vision, sound, and touch) as a teaching-learning strategy (Shams & Seitz, 2008).

The interface provides a graphical and sonic representation of trigonometric concepts, which are explored manipulating physical controllers using different gestures (sliding, pressing, rotating). Some controllers use a
custom design while others are recognizable elements, such as knobs, sliders and buttons. The graphical representation or Graphical User Interface (GUI) relies on the Unit Circle model, widely used in trigonometry (Kendal & Stacey, 1996) which is adequate to understand periodic functions and simplifies the calculation of sine and cosine based on an angle (Mesa & Goldstein, 2016). The Unit Circle is drawn on a screen, among other relevant graphical representations: sine and cosine values, the current angle, the cartesian plane, the sine wave and textual and numerical values where needed (Figure 3). Parameters of the Unit Circle are controlled mainly by what we call the “Rotary Wheel”, a thirty-two centimeter ring that can be turned in 360 degrees to control the angle that is drawn inside the Unit Circle on screen, consequently modifying trigonometric values (e.g. value of cosine). The rest of the controllers modify additional parameters (e.g. amplitude) and also enable navigating through scenes.

Sound has two roles: First, to highlight relevant information. For instance, when a special angle (0, 30, 45, 60 or 90 degrees, etc) is reached by turning the Rotary Wheel a beep sound is triggered, indicating the user that there’s something about that value that is worth exploring and analyzing; The second role is to represent trigonometric concepts as sound (e.g. sine waves represented as a variable tone to understand frequency).

Figure 3 : GUI of the interface (details), where the unit circle, angle, sine and cosine values can be visualized (left). Users can also interact with a visual representation of the sine wave by changing its frequency and amplitude which alters the properties of a tone that is heard through the speakers (right).

Intermediate Models and Prototype development

The design process relied on iterative prototyping and frequent user-testing, a standard method in the field of HCI for interface design that has the following proven benefits: enables to receive feedback from the participants in the early stages of development; guides designers to iterate diverse design options before committing to a particular path; and keeps the design process focused by abstracting the complexities of an integrated design into particular aspects to inform design decisions at each stage of development (Alperowitz, Weintraud, Kofler, & Bruegge, 2017; Dumas & Redish, 1999; Scarlatos, 2002).

The prototypes considered three dimensions based on the model proposed by Houde & Hill (1997): the “role” in respect of its use as a learning mediator; the “implementation” to define the technology and form of the interface; and the “look and feel” to consider the sensory and perceptual aspects.

In all the versions the common technology elements are: a physical device that integrates tangible controllers and sensors, a software that reads and process the data obtained from the sensors, and a 32-inch LCD screen that graphs the GUI and provides visual and sonic feedback based on the data readings (Figure 4).
Intermediate Models

Three intermediate models were designed and tested with 18 subjects, including students and professors from both the Design and Engineering Schools. Each session was twenty to thirty minutes long and included groups of one to four simultaneous participants. The sequence of Intermediate Models enabled to identify unexpected variables and helped resolve aspects such as ease of operation, collaboration dynamics, comprehension level of the content, enhancements of the physical space and progression of the script. For instance, in Model 1 the first iteration of the Rotary Wheel was a control detached from the main screen (Figure 5) and even though it was easily understood and operated, it still remained too close to the “mouse paradigm”. It was observed that users had a tendency to touch and indicate with their fingers directly on the screen, performing both deictic and iconic gestures (Rimé & Schiaratura, 1991), which led the team to integrate the Rotary Wheel on the screen for the following model (Figure 6).

Other decisions implemented included: modifying the position and thickness of the Rotary Wheel for better visibility of relevant information on screen, adding additional controls for the users (scene navigation, frequency and amplitude control, visualization of equations), removing/adding scenes and adjusting terminology and style used by the Facilitator.

The analysis of observations and questionnaires responded by users during this phase led the design of Prototype 1 (described in greater detail the following section). A selection of relevant opinions and observations extracted from a general matrix (where all data was transcribed) is presented as a summary in Tables 2 and 3.

Figure 4: Information flow diagram of the system.

Figure 5: Intermediate Model 1. Users interacting with the interface during a user-testing session.
Figure 6: Intermediate Model 2, the Rotary Wheel is now integrated into the screen.

Figure 7: Intermediate Model 3, user-testing sessions with Engineering professors to collect “expert” feedback.

Table 2: Summary of relevant observations collected during the sessions with Intermediate Models 1, 2 and 3.

<table>
<thead>
<tr>
<th>Observation highlights from sessions with intermediate models</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable 1 - Interaction</strong></td>
</tr>
<tr>
<td>● Interest in manipulating the interface increases as users understand the operation of the device.</td>
</tr>
<tr>
<td>● Users seek for precision in the numerical representation of angles and measurements.</td>
</tr>
<tr>
<td>● Users perform unexpected gestures, such as touching the screen to illustrate movement of the elements.</td>
</tr>
<tr>
<td>● The experience in standing position leads to a greater use of the body.</td>
</tr>
<tr>
<td>● Some students struggle with understanding some concepts when these are influenced by previous misconceptions.</td>
</tr>
</tbody>
</table>
Table 3: Selected opinions and suggestions from users during the testing sequence of intermediate models.

<table>
<thead>
<tr>
<th>Users’ relevant opinions - intermediate models</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What would you change to make the operation of the interface easier?</strong></td>
</tr>
</tbody>
</table>
| • “Add more manipulation controls to handle graphics”.
| • “That the two users could manipulate it [the interface] at the same time”.
| • “Improve accuracy.”
| • “Increase feedback through sounds.” |
| **Were you able to understand the contents presented?** |
| • “Yes, it is very well explained and helps to understand the relationships, it creates a mental map.”
| • “If they had explained this to me this way in [high] school, I think I would have understood way better.”
| • “Yes, but I missed an ending as a conclusion.” |
| **Was there any aspect that confused you?** |
| • “Trying to explain in simple words the meaning of sine and cosine.”
| • “Determine the radius of the circle.”
| • “None. Perhaps the precision of the values that appear on the screen.” |

<table>
<thead>
<tr>
<th>Professors’ suggestions - intermediate models</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regarding the interface</strong></td>
</tr>
</tbody>
</table>
| • Incorporate the possibility to visualize equations.
| • Explicitly display the quadrants in visual form.
| • Incorporate sound at key moments, for example when curves meet at the same point. |
| **Regarding content** |
| • Deepen the topic of the reflected angles and use the circle to explain Pi and radians.
| • Incorporate the connection between contents in an associative way. |

**Prototype 1**

Prototype 1 is the culmination of the first process of Intermediate Models and was tested with 31 subjects. The focus of this prototype was to validate the physical configuration, ideal number of participants per group and the progression of the content presented. Prototype 1 differs from the Intermediate Models in the sense that it is an integrated interface and learning experience, whereas the Intermediate Models were more focused on particular aspects and not necessarily on a fully functional device.

The interface consists of an LCD screen (where the trigonometric concepts are plotted) arranged horizontally and slightly angled. Several controls are built into a physical device that wraps the screen, organized by function in two distinct sections, albeit presented as a single integrated device. For this version, laser-cut agglomerated wood was used for its construction.

The main control is the Rotary Wheel (Figure 8), which controls the angle of the Unit Circle represented in the GUI (Figure 3). A series of secondary inputs consisting in buttons and knobs control additional system variables, grouped by function:

- **Navigation controls**: move forward and/or backward through scenes, change the angle manipulation mode (automatic or manual), activate and deactivate the display of “reflected angles”.
- **Parameter controls**: manipulate frequency and amplitude of the curve that graphs the periodic functions.
- **Debug controls (for Facilitator only)**: calibrate the position of the Rotary Wheel with the digitally drawn angle.
Inside the physical device there are a series of electronic components: two rotation sensors, a microcontroller, pressure sensors (buttons) and a linear potentiometer (slider). The rotation sensors measure angular position: one to measure the angle of the Rotary Wheel and the other to measure the angular position of a knob that controls the frequency. The microcontroller is an Arduino Leonardo that reads the voltage from the different sensors and sends a constant report to the serial port. The Arduino is connected to an external computer (via USB) that receives and interprets the data through a custom software developed in C++, using the OpenFrameworks library (https://www.openframeworks.cc/). The software is responsible for performing the graphic and auditory output that is displayed through the monitor (Figure 9).

Figure 8: Schematic render of Prototype 1, showing its configuration and distribution of controls.

Figure 9: Diagram of the technology and electronic components of Prototype 1.

Figure 10: User-testing session with Prototype 1.
Observation helped to conclude that three participants was the ideal number to enable a good field of view of the screen and making decisions based on consensus. An adequate number of users enable a more intimate social interaction (as they are physically close to each other) where nonverbal communication was often observed. It was also observed that on occasions participants had wrong or imprecise preconceptions of trigonometric concepts, which distorted or even prevented the acquisition of new knowledge (Ambrose et al., 2010). To address this issue, it was decided that for Prototype 2 the script would present concepts detached from their technical term at the beginning and introduced progressively throughout the session. For instance, during the first quarter of the experience, the word "sine" or "cosine" are intentionally replaced by "red bar" and "blue bar". This strategy enables participants to understand the meaning of concepts before associating them with their name, facilitating the student to incorporate the new knowledge with less resistance.

Table 4 summarizes the most relevant observations and suggestions collected during the testing sessions with Prototype 1.

<table>
<thead>
<tr>
<th>Relevant observations of Participants' testing sessions - Prototype 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable 1 - Interaction</strong></td>
</tr>
<tr>
<td>• The use of sound as a pedagogical resource surprises and attracts participants.</td>
</tr>
<tr>
<td>• The facilitator tends to invade the space of the student when having to operate certain buttons (such as scene navigation).</td>
</tr>
<tr>
<td>• It is necessary to determine strategies to encourage manipulation of the interface for students who adopt a more passive or observant attitude.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Suggestions from the users - Prototype 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What would you change or add to make the operation of the interface easier?</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Were you able to understand the contents presented?</strong></td>
</tr>
<tr>
<td><strong>Was there any aspect that confused you?</strong></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

**Prototype 2**

Prototype 2 is the culmination of the prototyping process. It was tested with 24 out of the 31 subjects that tested Prototype 1 (seven students could not attend the sessions). Both the content progression and the hardware is similar to Prototype 1, described in the previous section. The goal for this phase was to enhance the interaction experience through a better constructive fidelity and ease of operation. Thus, the main focus was on the redesign of the physical device.

All the electronic components (Arduino, sensors, cables, etc.) were placed hidden inside the structure to facilitate assembly and to achieve a cleaner look and feel of the device. The positions of the controls were reorganized, now providing more control possibilities to the user-end of the interface. A new control zone for
the Facilitator was implemented on the upper side of the interface (where the Facilitator stands) to keep the screen unblocked at all times (Figure 11 and 12). Parts of the Rotary Wheel and the gear connected to the rotation sensor (rotary encoder) were now made out of acrylic (Plexiglas), which performed better under mechanical stress and reduces friction, therefore achieving more precision and smoothness when operated.

Figure 11: Schematic render of Prototype 2, showing its configuration and distribution of controls.

The qualitative analysis of data obtained during user testing with the Intermediate Models and Prototype 1 also impacted on decisions regarding the physical space (Jetter et al., 2014). In the previous versions participants were seated in chairs, this limited the users to exchange roles as they had a restricted displacement around the interface. In Prototype 2, a stand for the interface was designed to obtain a suitable height and angle when standing in an upright position, which resulted in a more active, collaborative and fluid interaction, making it easier for participants to take turns when operating certain controls (Figure 12). Overall it was observed that the implemented design changes enhanced the experience as users were more focused on the content and less on the operation of the interface.

Figure 12: Prototype 2 in use, now in standing position.

Learning assessment results

Pre-Test Results

The Pre-Test was designed and conducted at the beginning of the process of the study, based on the Diagnostic Evaluation of first-year Engineer students (Table 1). It consisted of 12 questions, each one was worth 1 point. It covered the five thematic Modules described previously and included a written introduction at the beginning of the test to even the minimum knowledge level needed to answer the test. It was taken by 119 students: 65 from Design and 54 from Engineering School.

Results of the Pre-Test were low. As seen in Table 5, of a maximum of 12 total points, in Engineering the mean score was 4.35 points (36% yield), and in Design of 2.44 points (20% yield).

Table 5: Mean scores and performance obtained by Design and Engineering students in the Pre-Test.
The overall performance in Design was lower than in Engineering (which was expected). However, when analyzing the results by thematic module and by question, the performance curves were very similar in both careers, showing that the difficulties are coincident and that the level of understanding of basic concepts of trigonometry was transversally low (see Figure 13).

Figure 13: Pre-Test results by Thematic module (left) and by question (right). As observed, the performance curves are similar in both careers.

Post-Test Results

The Post-Test was identical to the Pre-Test and the same data collection and analysis methods were used. The Post-Test was taken only by the Experimental Group (n = 24, Design and Engineering students) who attended two sessions with the interface (Prototypes 1 and 2). Additionally, a Design Control group (n = 16) took the Post-Test without participating in any session with the interface and without receiving formal instruction.

When comparing the results of the Pre and Post-Test, a significant improvement was observed in the Experimental Group (statistically confirmed by a two sample t-test and a paired sample t-test). In Design, the increase was of 5.0 points, and in Engineering 3.9 (4.46 points when considering both careers). It is noteworthy that even though Engineering students were exposed to additional formal instruction in trigonometry during the lapse of this study (as opposed to Design students) their score improvement was comparatively lower than the score improvement of Design students. Further studies are needed to assess with greater detail the influence of the interface in the case of students receiving additional trigonometry instruction, but the current analysis supports that the interface does have an impact in students’ performance overall.

Figure 14: Comparison of Pre-Test and Post-Test results in the Experimental Group (left). Distribution graph plotting the scores obtained by the Experimental Group on the Pre-Test and Post-Test, disaggregated by discipline (right).
Table 5 shows the comparative performance per question in the Pre-Test and Post-Test for the Experimental Group as a whole. After going through the experience with the interface, performance increased from 34.4% to 71.6%, yielding a differential of +37.1%, which indicates that the experience positively impacted learning.

Table 5 : Performance per question on Pre-Test and Post-Test, both careers.

<table>
<thead>
<tr>
<th>Question number</th>
<th>Pre-Test performance (%)</th>
<th>Post-Test performance (%)</th>
<th>Differential (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.5</td>
<td>91.4</td>
<td>+ 78.9</td>
</tr>
<tr>
<td>2</td>
<td>17.5</td>
<td>91.4</td>
<td>+ 73.9</td>
</tr>
<tr>
<td>3</td>
<td>39.2</td>
<td>96.4</td>
<td>+ 57.3</td>
</tr>
<tr>
<td>4</td>
<td>45.0</td>
<td>92.9</td>
<td>+ 47.9</td>
</tr>
<tr>
<td>5</td>
<td>27.5</td>
<td>47.1</td>
<td>+ 19.6</td>
</tr>
<tr>
<td>6</td>
<td>35.0</td>
<td>77.9</td>
<td>+ 42.9</td>
</tr>
<tr>
<td>7</td>
<td>62.5</td>
<td>77.9</td>
<td>+ 15.4</td>
</tr>
<tr>
<td>8</td>
<td>61.7</td>
<td>78.6</td>
<td>+ 16.9</td>
</tr>
<tr>
<td>9</td>
<td>40.0</td>
<td>38.6</td>
<td>- 1.4</td>
</tr>
<tr>
<td>10</td>
<td>18.3</td>
<td>65.7</td>
<td>+ 47.4</td>
</tr>
<tr>
<td>11</td>
<td>30.8</td>
<td>70.7</td>
<td>+ 39.9</td>
</tr>
<tr>
<td>12</td>
<td>23.3</td>
<td>30.0</td>
<td>+ 6.7</td>
</tr>
<tr>
<td>Average</td>
<td>34.4%</td>
<td>71.6%</td>
<td>+ 37.1%</td>
</tr>
</tbody>
</table>

The performance increase in the Design Experimental Group (n=10) is particularly noteworthy when compared to the Control Group (n=16) of the same discipline. Even though both groups took the Pre-Test and Post-Test, the Experimental Group had an increase of +5.0 points, while the Control Group only increased +0.64 points (Figure 15). This difference strengthens the conclusion that the performance increase is mostly attributable to the interface, and that being exposed to the same test twice did not have a relevant impact on their scores.

Figure 15 : Score obtained in Pre-Test and Post-Test by the Experimental and Control groups of the Design career. It can be seen that the Control Group does not show a significant improvement, which contrasts with those exposed to the interface that increased their score by +5.0 points.
Discussion and Conclusions

The main goal of this study was to explore the potential of TUIs as a tool to teach and learn basic concepts of trigonometry. Although the fact that trigonometry concepts are hard to teach and learn was justified by the literature, the specific case of students at Universidad del Desarrollo in Chile used as a case study in this research is not generalizable to the Chilean or global context. Nevertheless, results from the study endorse the potential of TUIs as pedagogical tools to enhance learning. Students who participated in the sessions with the interface improved their performance, but most importantly, data analysis showed that their cognitive process was positively influenced individually and collaboratively by experiencing the interface (Jetter et al., 2014). A limitation to these results was not counting with a formal Control Group in Engineering who received formal instruction but was not exposed to the use of the interface. This was due to the loss of Engineering students: some had taken the Pre-Test, but left the career after the first semester, and others failed their first semester and had intensive trigonometry classes during the second semester where they covered advanced trigonometry concepts.

Using multiple senses to transfer the content contributed to diversify the access routes to the information during the learning experience. For example, when participants used the Rotary Wheel, they could see how the angle and values of sine and cosine changed, making the abstract visible in a concrete way. Manipulating a physical representation of digital information proved to be beneficial for students to internalize abstract concepts in concrete terms (Lakoff & Núñez, 2000). Being able to repeat an action as many times as necessary to understand the phenomenon enabled students to reflect and agree on opinions, activating their learning process and making it more participatory. In addition, the use of a simple and non-technical language during the experience, facilitated the construction of metaphors that serve to achieve understanding of abstract notions of mathematics (Font et al., 2010).

It was inspiring to observe the process of co-creation of knowledge among participants from different disciplines when they discussed while using the tangible manipulative attributes of the interface. Students were able to understand concepts that they did not understand before, corroborate or question their previous knowledge, and learn together using the interface.

At the actual point of development, the project could not entirely resolve the design principle proposed by Resnick et al. (2005), to offer low limits, high ceilings and wide walls. Today, the interface offers ideal low limits for novices and partially offers a wide range of exploration possibilities (wide walls) but does not enable more sophisticated interactions (high ceilings). Nevertheless, the research team agrees that the interface in the future could evolve into an educational tool to approach teaching-learning of mathematics for both teachers and students. By appealing more to the understanding of the concepts and less to the direct resolution of mathematical problems, researchers believe that the interface can be a useful ally to facilitate the entry to the abstract approach needed to develop more complex skills in advanced mathematics.

References


